

2.2 MODELING OF HEAT TRANSFER

The overall heat transfer coefficient U depends on the distribution of thermal resistances in the exchanger. If the exchange surfaces are equal for both fluids:

$$\frac{1}{U} = \frac{1}{h_h} + \frac{e}{\lambda} + \frac{1}{h_c} \quad (2.2.1)$$

The values of convection coefficients h_h and h_c are based on fluid thermophysical properties and exchange configurations, convective regimes being strongly dependent on the flow velocity.

They can be obtained from correlations giving the value of Nusselt number $Nu = \frac{h d_h}{\lambda}$, depending on Reynolds $Re = \frac{\rho C d_h}{\mu}$ and Prandtl $Pr = \frac{\mu c_p}{\lambda}$ numbers.

In implementations proposed in Thermoptim, we take into account the thermal resistance of the wall (as well as that of the fins if they exist), and calling S_c and S_f the total exchange areas of hot and cold sides, we relate these two surfaces to a common reference (or primary) surface S or A , called exchanger surface. For this, we introduce two "surface factors" f_h and f_c , such as $S_h = f_h A$, and $S_c = f_c A$. If there are no extended surfaces, $f_h = f_c = 1$.

The calculation of U is ultimately done by:

$$\frac{1}{U} = \frac{d_{hh}}{\lambda_h Nu_h} + \frac{e}{\lambda} + \frac{d_{hc}}{\lambda_c Nu_c}$$

2.2.1 EXTENDED SURFACES

If there are extended surfaces, one takes as a reference surface that without extended surfaces, and calculates a surface factor greater than 1 for the other (Figure 2.2.1). Taking into account the existence of fins, $\eta_{0,h}$ and $\eta_{0,c}$ being the overall effectiveness of fins hot and cold sides, we have:

$$\frac{1}{U_h A_h} = \frac{1}{A_h \eta_{0,h} h_h} + \frac{e}{A_w \lambda} + \frac{1}{A_c \eta_{0,c} h_c} \quad (2.2.2)$$

with A_h and A_c total exchange areas warm and cold sides, A_w exchanger wall surface, and $\eta_{0,h}$ and $\eta_{0,c}$ overall fin effectiveness on warm and cold sides (see section 5.1.3).

There is also a cold side global exchange coefficient U_c , with of course:

$$U_c A_c = U_h A_h$$

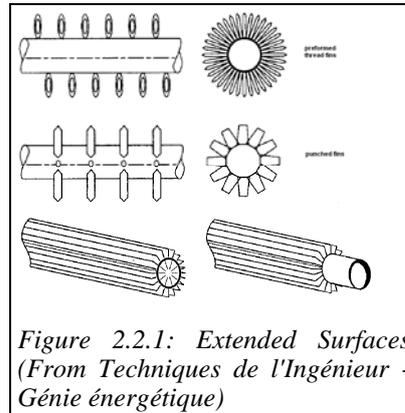


Figure 2.2.1: Extended Surfaces
(From *Techniques de l'Ingénieur - Génie énergétique*)

2.2.2 CALCULATION OF REYNOLDS AND PRANDTL NUMBERS

$Re = \frac{\rho C d_h}{\mu}$ can be expressed directly from fluid mass flow, fluid free flow area A_c , and mass velocity $G = \rho C$.

Since $\dot{m} = G A_c$

$$Re = \frac{G d_h}{\mu} = \frac{\dot{m} d_h}{\mu A_c} \quad (2.2.3)$$

Knowing mass flow, you have to give the free flow area A_c and the hydraulic diameter d_h .

For gases, the Prandtl number varies slightly depending on temperature and remains between 0.7 and 0.75. It can therefore be considered constant without committing an error. For liquids, it is imperative to consider temperature in the computation of Pr. This is obviously what is done in Thermoptim, where thermophysical properties of substances are calculated accurately.

2.2.3 CALCULATION OF THE NUSSELT NUMBER

The Nusselt number is almost always given by the following equation:

$$Nu = C_1 Re^a Pr^b (\mu/\mu_p)^c \quad (2.2.4)$$

where C_1 , a , b , and c are constants. Coefficients C_1 , a and b vary depending on configurations encountered, and the additional term to account for variations in viscosity is often neglected.

The viscosity correction is made at the wall temperature. However, it is not calculated in Thermoptim. Initially, the solution is to estimate it as being equal to the average temperature of both fluids, but this is only valid in first approximation, and in the case of a multizone heat exchanger, this can lead to discrepancies.

In general, the Reynolds number exponent is between 0.5 and 0.8, and that of the Prandtl number between 0.33 and 0.4. You can find in the literature values of correlations for very special finned geometries.

The flow regime has a significant influence on the value of Nu. Where Re is less than 2,000, the regime is laminar, and if Re is greater than 5,000, it is turbulent, with a transition zone, these boundaries not being perfectly stable.

For established laminar flow, $Nu = \text{Const}$, the value being between 4 and 8 or so, depending on the shape of the tube and the heat exchange mode (constant temperature or flux).

2.2.3.1 Inside tubes

Inside tubes, the formula most used is that of Mac Adams and Dittus-Boettler:

$$Nu = 0,023 Re^{0,8} Pr^{0,4} \quad (2.2.5)$$

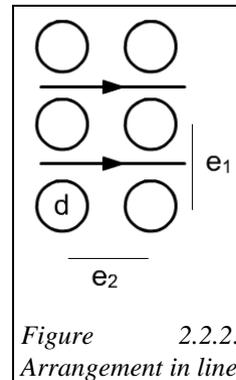


Figure 2.2.2:
Arrangement in line

2.2.3.2 Flow perpendicular to tubes

For flows perpendicular to tubes, the arrangement of tubes influences the exchange coefficients, and several formulas exist, such as Colburn:

$$Nu = 0,33 K_p K_r Re^{0,6} Pr^{0,33} \quad (2.2.6)$$

Coefficients K_p and K_r depend on the arrangement and relative values of e_1 , e_2 and d (K_p of the number of rows, and K_r of the network geometry and Re). At first, not to overcomplicate things, we have neglected the influence of these two terms in external classes provided with Thermoptim, taking them both equal to 1.

2.2.3.3 Finned coils

For finned coils, predictive correlations are relatively complex. However ultimately, things are simplified, and it can be shown that simple relationships such as (2.2.7) can be used.

$$h = K_1 Re^{0,77} = K_2 G^{0,77} = K_3 V^{0,77} \quad (2.2.7)$$

Just identify coefficient K based on the parameter chosen to obtain a simple relationship characterizing heat exchange air side.

2.2.3.4 Two-phase exchange

For two-phase exchange, things are much more complicated. Most accurate methods take into account several zones depending on the vapor quality and a related quantity, called void fraction. Some correlations calculate h from an implicit relationship, while others prevent this complication.

In correlations used for two-phase exchange, Nusselt is often calculated as the product of a liquid Nusselt by an amplification factor, Nu_{liq} being calculated from the MacAdams correlation, valid inside a tube. It is a legitimate question whether this calculation remains valid for other types of heat exchangers including plate heat exchangers. Can we especially recalculate Nu_{liq} for the configuration chosen, and keep the amplification factor? As discussed below, the tests suggest not.

formulation for condensation outside horizontal tubes

We selected correlation given by W. Levy, section 1.2.2 of article B1540 from Techniques de l'Ingénieur:

$$Nu = \frac{3,022 d_h}{\lambda} \left[\frac{\lambda^3 H \rho^3 g}{d_h \mu \Delta \theta} \right]^{0,25} \quad (2.2.8)$$

formulation for condensation in tubes

We selected correlation of Shah (1979), modified by Bivens (1994):

$$Nu = Nu_{10} F = 0,023 Re_{10}^{0,8} Pr_{10}^{0,4} \left[(1-x)^{0,8} + \left(\frac{3,8 x^{0,76} (1-x)^{0,04}}{\left(\frac{P}{P_c}\right)^{0,38}} \right) \right]$$

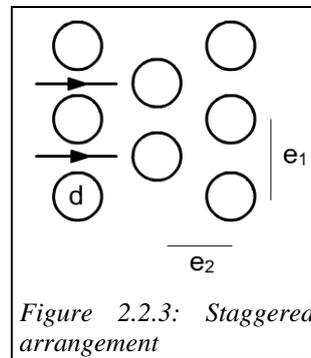


Figure 2.2.3: Staggered arrangement

$$Nu = Nu_{Shah} \left[0,78738 + 6187,89 G^{-2} \right]$$

In this expression, the calculation of Nu_{10} must be done with the total mass flow \dot{m} .

This relationship is applied by averaging the F factor in the range of variation of x, which simply asks to calculate average values for the integrated functions of x. The following equation was identified numerically to obtain a simple expression of F averaged:

$$\int x^{0,76} (1-x)^{0,04} dx = -0,0155 x^3 + 0,1669 x^2 + 0,1083 x - 0,0065$$

$$\int (1-x)^{0,8} dx = -0,5555556 (1-x)^{1,8}$$

formulation for the evaporation in tubes

We selected the Gungor and Winterton correlation (1987):

$$Nu = Nu_{liq} F$$

$$Nu = 0,023 Re_{liq}^{0,8} Pr_{liq}^{0,4} \left[\left(1 + 3000 Bo^{0,86} + 1,12 \left(\frac{x}{1-x} \right)^{0,75} \left(\frac{v_v}{v_l} \right)^{0,41} \right) \right]$$

In this expression, the calculation of Nu_{liq} must be done with the liquid phase flow alone $(1-x) \dot{m}$.

This relationship is applied by averaging the F factor in the variation range of x, taking into account the flow of the liquid phase alone. The following equation was identified numerically to obtain a simple expression of F averaged:

$$\int \left(\frac{x}{1-x} \right)^{0,75} (1-x)^{0,8} dx = -0,0079 x^4 - 0,131 x^3 + 0,6128 x^2 + 0,0651 x - 0,0054$$

$$\int (1-x)^{0,8} dx = -0,5555556 (1-x)^{1,8}$$

The expression of Bo, boiling number is given below in general, then for a tube, q_{th} being the flux density, A_{ext} the outer surface and G the mass velocity:

$$Bo = \frac{q_{th}}{G L_v}, \text{ which can be approximated as follows for a tube:}$$

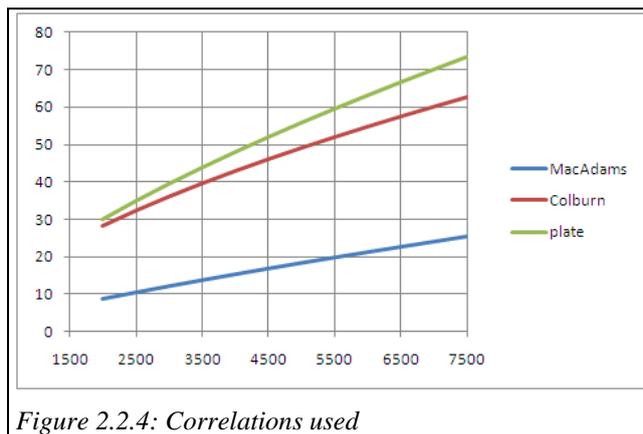


Figure 2.2.4: Correlations used

$$Bo = \frac{m L_v}{A_{ext} G L_v} \frac{1}{G L_v} = \frac{G A_c L_v}{A_{ext} G L_v} \frac{1}{G L_v} = \frac{A_c}{A_{ext}} = \frac{d_h}{4 L}$$

2.2.3.5 plate heat exchangers

For plate heat exchangers, exponent b is almost always 1/3 and c 0.14 for Muley and Manglik (1999) and 0.17 for Kumar (1984).

For chevron of angles $\leq 30^\circ$ Kumar (1984) gives 0.348 for C_h and 0.663 for a. Alfa Laval provides $C_h = 0.142$, $a = 0.72$, $b = 0.37$ and $c = 0.108$ for the A20 exchanger. Manglik and Muley (1999) propose a more complex formula for chevron angles $\beta = 90 - \alpha$ from 30° to 60° , and surface factors μ from 1 to 1.5 for $Re > 10^3$.

$$Nu = \left[0.2668 - 0.00696\beta + 7.244 \times 10^{-5} \beta^2 \right] \times \left[2078 - 5084\mu + 41.16\mu^2 - 10.51\mu^3 \right] \\ \times Re^{[0.728 + 0.0543 \sin[(\pi\beta/45) + 3.7]]} Pr^{1/3} (\eta/\eta_w)^{0.14}$$

Figure 2.2.4 shows the importance of the influence of the correlation used on Nu value as a function of Re for $Pr = 0.7$. Correlations used are listed hereafter.

- inside tubes (Mac Adams): $Nu = 0.023 Re^{0.8} Pr^{0.4}$
- outside tubes (Colburn) : $Nu = 0.33 Re^{0.6} Pr^{0.33}$
- plate with viscosity correction: $Nu = 0.2 Re^{0.674} Pr^{0.33} (\mu/\mu_p)^{0.14}$

Figures 2.2.4 and 2.2.5 show the influence of correlation parameters on the Nusselt number, and on the exchange coefficients.

As expected, we note that the correlation for the plate heat exchanger leads to U values larger than those of Mac Adams and Colburn. However, it is not the only one to be proposed (Ayub, 2003).

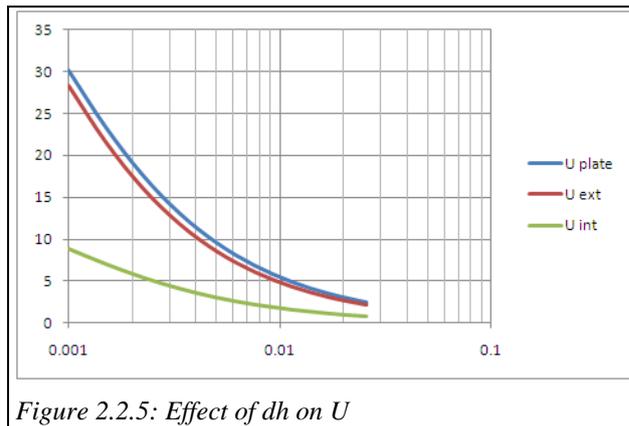


Figure 2.2.5: Effect of dh on U

2.2.4 CALCULATION OF MULTI-ZONE EXCHANGERS

The calculation of U by $\frac{1}{U} = \frac{d_{hh}}{\lambda_h Nu_h} + \frac{e}{\lambda} + \frac{d_{hc}}{\lambda_c Nu_c}$ is only valid for a simple exchanger, through which flow two fluids with constant thermophysical properties.

For multizone exchangers such as evaporators and condensers for refrigeration machines, the calculation of U is more difficult, because the overall exchanger is actually a heat exchanger in series or series-parallel because of the linkages between the three sub exchangers-liquid (l), liquid-vapor (lv) and vapor (v), which share the total area of the exchanger. Only the total area being known, it is necessary to compute simultaneously the three parts of the exchanger to find the solution.